# Technical Note 16 – Equivalent Static Method

# Contents

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## 1 Introduction

A buried continuous PVC/Welded steel pipeline is designed to dispatch water at a pressure of 650 kPa. The pipe used for this analysis is either DN450 Steel pipe or DN300 Class C PVC pipe.

For typical pipeline trench profile shown in Figure 1-1:

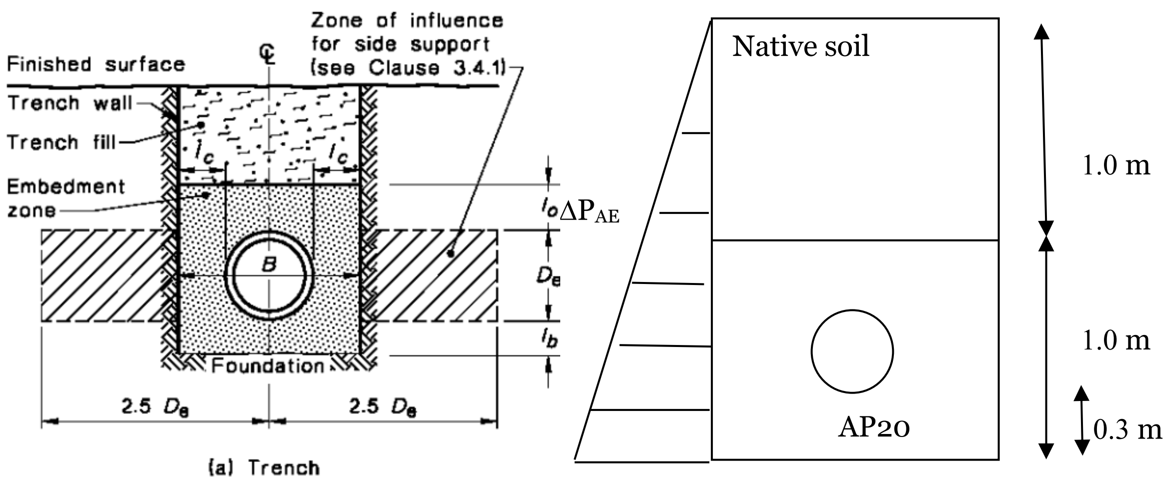


**Figure 1-1: Typical pipeline trench**

The seismic hazards which can directly cause catastrophic failures to pipelines can be classified as

- Permanent Ground Displacement (PGD)
  - Longitudinal permanent ground deformation
  - Transverse permanent ground deformation
  - Landslide
- Buoyancy due to liquefaction
- Fault crossing
- Seismic wave propagation

# 2 Operational Strain in the Pipeline

Romberg-Osgood parameter $n = 9$
Rameberg-Osgood parameter $r = 10$
Working pressure = 650 kPa

Continuous pipeline

- Pipe strain due to internal pressure
  - The longitudinal stress induced in the pipe due to internal pressure will be:

$$S_p = \frac{PD\mu}{2t} = \frac{650000 \times 0.506 \times 0.3}{2 \times 0.004} = 12.33 \times 10^6\, N/m^2 = 12.33MPa$$

  - Using Ramberg-Osgood's stress-strain relationship, the longitudinal strain in the pipe will be:

$$\varepsilon_p = \frac{S_p}{E}\left[1 + \frac{n}{1+r}\left(\frac{S_p}{\sigma_y}\right)^r\right] = \frac{12.33 \times 10^6}{2 \times 10^{11}}\left[1 + \frac{9}{10+1}\left(\frac{12.33 \times 10^6}{358 \times 10^6}\right)^{10}\right] = 0.000062 = 0.0062\%$$

# 3 Seismicity

Based on NZS1170.5:2004:

$$C(T) = c_h(T)ZRN(T,D)$$

Where

$c_h(T)$ = the spectral shape factor
$Z$= the hazard factor
$R$ = the return period factor
$N(T,D)$ = the near-fault factor

Table 3-1 summarises the design PGAs for Auckland, Wellington and Christchurch based on different soil subclass, and annual probability of exceedance (AEP).

**Table 3-1: Design PGAs for Auckland, Wellington and Christchurch (NZS1170.5)**

| LOCATION | $c_h(T)$ | | | Z | R | | N(T,D) | PGA | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Class A | Class C | Class D & E | | AEP 1/50 | AEP 1/100 | | Soil Class A | | Soil Class C | | Soil Class D & E | |
| | | | | | | | | AEP 1/50 | AEP 1/100 | AEP 1/50 | AEP 1/100 | AEP 1/50 | AEP 1/100 |
| Auckland | 1.89 | 2.36 | 3.00 | 0.13 | 0.25 | 0.5 | 1.0 | 0.06 | 0.12 | 0.08 | 0.16 | 0.09 | 0.18 |
| Wellington | 1.89 | 2.36 | 3.00 | 0.40 | 0.25 | 0.5 | 1.0 | 0.19 | 0.38 | 0.27 | 0.54 | 0.35 | 0.70 |
| Christchurch | 1.89 | 2.36 | 3.00 | 0.22 | 0.25 | 0.5 | 1.0 | 0.10 | 0.20 | 0.13 | 0.26 | 0.17 | 0.34 |

# 4 Vertical Uplift

The maximum soil resistance per unit length of the pipeline in vertical uplift can be calculated as

$$Q_u = N_{cv}cD + N_{qv}\overline{\gamma}HD$$

Where

$N_{cv}$ = Vertical uplift factor for clay (0 for = 0 )

$N_{qv}$ = Vertical uplift factor for sand (0 for $\emptyset = 0^0$ )

$N_{cv} = 2\left(\frac{H}{D}\right) \leq 10$ for $\left(\frac{H}{D}\right) \leq 10$, and

$N_{qv} = \left(\frac{\emptyset H}{44D}\right) \leq N_q$

Soil conditions:

Granular: $\emptyset = 32^o$

Cohesive soil: $\emptyset = 28^o$ , $c = 5\ kPa$

The mobilizing displacement of the soil,$\Delta_{qu}$ at $Q_u$ can be taken as

(a) $0.01H$ to $0.02H$ for dense to loose sands$\leq 0.1D$, and

(b) $0.1H$ to $0.2\ H$ for stiff to soft clay$\leq 0.2D$.

**Table 4-1: Summary of uplift results**

| | DN450 STEEL /GRANULAR | DN450 STEEL/COHESIVE SOIL | DN300 PVC/Granular | DN300 PVC/COHESIVE SOIL |
|---|---|---|---|---|
| $N_{cv}$ | 5.8 | 5.8 | 8.7 | 8.7 |
| $N_{qv}$ | 2.1 | 1.8 | 3.2 | 2.8 |
| $\gamma(kN/m^3)$ | 19 | 18 | 19 | 18 |
| $Q_u(kPa)$ | 25 | 60 | 25 | 35 |

# 5 Vertical Bearing

The maximum soil resistance per unit length of pipeline in vertical bearing can be calculated as

$$Q_d = N_c c D + N_q \overline{\gamma} H D + N_\gamma \gamma \frac{D^2}{2}$$

Where

$N_c$, $N_q$, $N_\gamma$ are bearing capacity factors (Figure 5-1 below)
$\gamma$ is total unit weight of soil
$c$ is soil cohesion
$D$ is outside diameter of pipe
$H$ is the depth of soil above the centre of the pipeline
$\overline{\gamma}$ is effective unit weight of soil

Soil conditions:

Granular: $\emptyset = 32^o$
Cohesive soil : $\emptyset = 28^o$ , $c = 5\ kPa$

**Table 5-1: Summary of vertical bearing results**

| | DN450 STEEL /GRANULAR | DN450 STEEL/COHESIVE SOIL | DN300 PVC/Granular | DN300 PVC/COHESIVE SOIL |
|---|---|---|---|---|
| $N_c$ | 35 | 26.5 | 35 | 26.5 |
| $N_q$ | 23.5 | 15 | 23.5 | 15 |
| $N_\gamma$ | 26.2 | 12.5 | 26.2 | 12.5 |
| $\gamma (kN/m^3)$ | 19 | 18 | 19 | 18 |
| $Q_d (kPa)$ | 630 | 250 | 630 | 250 |

The mobilizing soil displacement, $\Delta_{qd}$, at $Q_d$ can be taken as

$0.1D$ for granular soils, and $0.2D$ for cohesive soils.

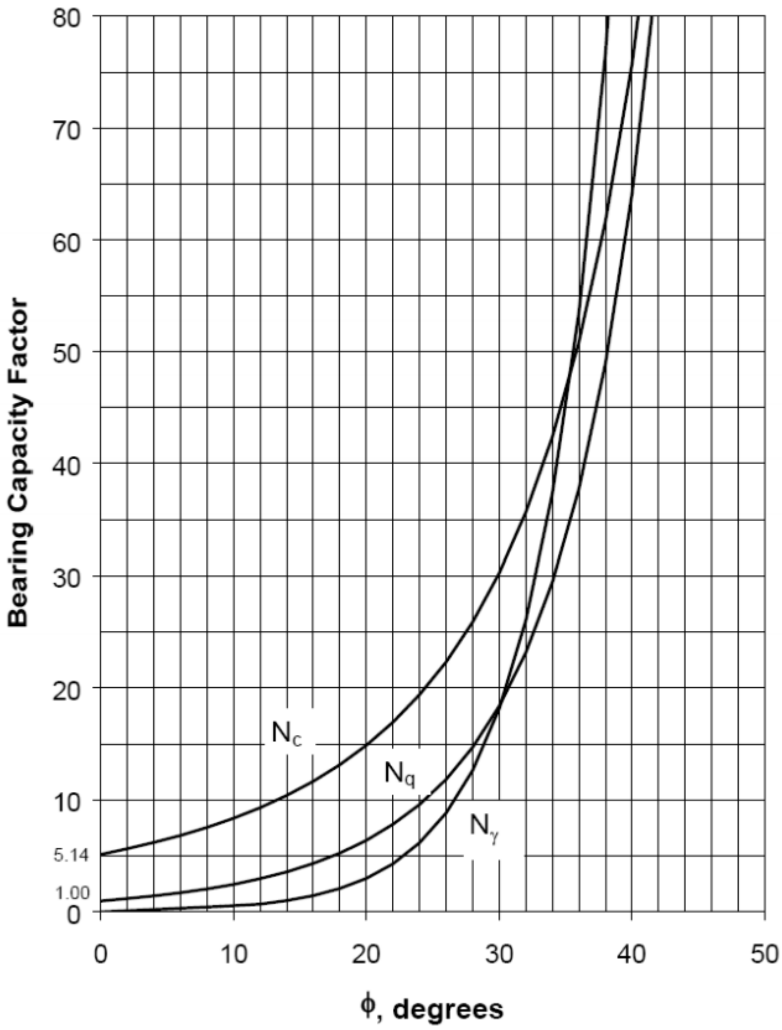


**Figure 5-1: Bearing capacity factors of soils of different soil frictions (ALA 2001)**

# 6 Design Load for Permanent Ground Deformation (PGD)

When the stress-strain relationship for the pipe material is not defined, it may be approximated by Ramberg- Osgood's relationship as:

$$\varepsilon = \frac{\sigma}{E}\left[1 + \frac{n}{1+r}\left(\frac{\sigma}{\sigma_y}\right)^r\right]$$

Where

$\varepsilon$ = Engineering strain
$\sigma$ = Engineering strain
$E$ = Initial Young's modulus
$\sigma_y$ = Engineering strain
$n, r$ = Ramberg –Osgood parameters (refer to Table 3.7.4).

For DN450 Steel pipe,

The stress in the pipeline is

$$\sigma = \frac{E\varepsilon}{\left[1 + \frac{n}{1+r}\left(\frac{\sigma}{\sigma_y}\right)^r\right]}$$

Rameberg-Osgood parameter $n = 9$
Rameberg-Osgood parameter $r = 10$

## 6.1 Parallel Crossing (Longitudinal PGD)

The expected amount of permanent ground movement parallel to pipe axis is $\delta^l = 2m$

Length of the permanent ground displacement zone is $100\ m$

The design ground movement = $\delta^l_{design} = \delta^l \times I_p$

Where

$I_p$ = Important factor

For pipeline of class III (1/475 year return period, APE in 50 years of 10%), $I_p = 1.0$

**Table 6-1: Important factor for different classes of pipeline ($I_p$)**

| CLASS OF PIPELINE | WAVE PROPAGATION | FAULTING | TRANSVERSE AND LONGITUDINAL PGD | LANDSLIDE |
|---|---|---|---|---|
| I | 1.5 | 2.3 | 1.5 | 2.6 |
| II | 1.25 | 1.5 | 1.35 | 1.6 |
| III | 1.0 | 1.0 | 1.0 | 1.0 |
| IV | - | - | - | - |

Hence the design longitudinal ground displacement is = $\delta^{l}_{design} = 2\times1.35 = 2.7m$

According to Case 1:

The amount of ground movement ($\delta^{l}_{design}$) is considered to be large the pipe strain is controlled by the length (L) of permanent ground deformation zone. The peak pipe strain (tensile/compressive) for this case is calculated as

$$\varepsilon_a = \frac{t_u L}{2\pi D t E}\left[1 + \frac{n}{1+r}\left(\frac{t_u L}{2\pi D t \sigma_y}\right)^r\right]$$

Where

$t_u$= Maximum axial soil force per unit length of pipe for specific site conditions

The expression of $t_u$is

$$t_u = \pi D c \alpha + \pi D H \overline{\gamma}\left(\frac{1+K_0}{2}\right) tan\delta'$$

Where:

$D$ =Diameter of pipe = 0.457 m
$c$ = Coefficient of cohesion = 30 kPa
$\alpha$ = Adehesion Factor = $0.608 - 0.123c - \frac{0.274}{c^2+1} + \frac{0.695}{c^3+1} = 0.9964$
$H$ = Soil cover above the centre of the pipeline = 1.3 m
$\overline{\gamma}$ = Effective unit weight of soil = 18000 $N/m^3$

Interface angle of friction between soil and pipe = $\delta' = f\emptyset$

Where,

$f$ = friction factor = 0.7 for smooth steel pipe
$\emptyset$ = Internal friction angle of soil = $30^o$

Hence, $\delta' = f\emptyset = 0.7 \times 30^o = 21^o$

$K_0$ = Coefficient of soil pressure at rest = $1 - sin30^o = 0.5$

Hence,

$$t_u = \pi \times 0.506 \times 30000 \times 0.9964 + \pi \times 0.506 \times 1.3 \times 18000 \times \frac{1+0.5}{2} tan21^o = 52585\, N/m = 53\ kN/m$$

Hence,

$$\varepsilon_a = \frac{52585 \times 100}{2 \times \pi \times 0.452 \times 0.004 \times 2 \times 10^{11}} \left[1 + \frac{9}{1+10}\left(\frac{52585 \times 100}{2 \times \pi \times 0.452 \times 358 \times 10^6}\right)^{10}\right] = 0.0023 = 0.23\%$$

According to Case 2:

The length (L) of permanent ground deformation zone is large, and the pipe strain is controlled by the amount of ground movement ($\delta^l_{design}$). The peak pipe strain (tensile/compressive) for this case is calculated as

$$\varepsilon_a = \frac{t_u L_e}{2\pi DtE}\left[1 + \frac{n}{1+r}\left(\frac{t_u L_e}{2\pi Dt\sigma_y}\right)^r\right]$$

Where $L_e$ can be obtained from the following relationship

$$\delta^l_{design} = \frac{t_u L_e}{2\pi DtE}\left[1 + \left(\frac{2}{2+r}\right)\left(\frac{n}{1+r}\right)\left(\frac{t_u L_e}{2\pi Dt\sigma_y}\right)^r\right]$$

Hence, the effective length of the pipeline is calculated as $L_e = 450\ m$

Hence,

$$\varepsilon_a = \frac{52585 \times 450}{2 \times \pi \times 0.452 \times 0.004 \times 2 \times 10^{11}} \times \left[1 + \frac{9}{1+10} \times \left(\frac{52585 \times 450}{2 \times \pi \times 0.452 \times 358 \times 10^6}\right)^{10}\right] = 0.0104 = 1\%$$

The design strain in the pipe is taken as the least value between the two cases =0.23% (tensile/compressive)

The operational strain in the pipeline = $\varepsilon_{oper} = 0.000062$

Hence,

Total tensile strain in the pipeline = 0.0023+0.000062 = 0.0024

And total compressive strain = 0.0023 -0.000062 = 0.0022

The limiting strain in tension of steel pipe for permanent ground deformation is 3% = 0.03

Hence, the total strain in the pipe due to longitudinal strain is less than the allowable strain.

## 6.2 Transverse crossing (Transverse PGD)

The expected amount of transverse permanent ground deformation ($\delta^t$)= 2m

The design transverse ground displacement = $\delta^t_{design} = \delta^t \times I_p = 2\times1.0 = 2m$

The maximum bending strain in the pipe is calculated as the least value two

i) $\delta_b = \pm\frac{\pi D\delta^t_{design}}{W^2} = \pm\frac{\pi\times0.506\times2}{40^2} = \pm0.0006325$, and

ii) $\delta_b = \pm\frac{P_u W^2}{3\pi EtD^2}$

$$P_u = N_{ch}cD + N_{qh}\overline{\gamma}HD$$

Where

$N_{ch}$ = Horizontal bearing factor for clay = 5.9
$N_{qh}$ = Horizontal bearing for sandy soil = 6.7

Hence,

$$P_u = 5.9\times30000\times0.506 + 6.7\times18000\times1.3\times0.506 = 168893\,N/m = 169\,kN/m$$

Hence,

$$\delta_b = \pm\frac{168893\times40^2}{3\times\pi\times2\times10^{11}\times0.004\times0.506^2} = 0.14$$

Hence, the maximum strain induced in the pipeline due to transverse PGD is taken as $\varepsilon_{seismic} = \pm0.0006325$ (tensile/compressive)

The operational strain in the pipeline =$\varepsilon_{oper} = 0.000062$

Hence,

Total longitudinal strain in the pipe in tension = 0.0006325+0.000062=0.0006945

Total longitudinal strain in the pipe in compression = 0.0006325- 0.000062=0.0005705

The allowable strain in tension for permanent ground deformation = 3% = 0.03

The allowable strain in compression for steel pipe is = $\varepsilon_{cr-c} = 0.175\frac{t}{R} = 0.175\frac{0.004}{0.253} = 0.0028$

The total strain in pipe due to transverse PGD is less than the allowable strain for both tension and compression.

# 7 Buoyant Force on Pipeline

The net upward force per unit length of pipeline due to buoyancy can be calculated as:

$$F_b = W_s - \left[W_p + W_c + (P_v - \gamma_w)D\right]$$

Where

$W_s$ = total weight of soil displaced by pipe per unit length
$W_p$ =weight of pipe per unit length
$W_c$= Weight of pipe content per unit length
$P_v$= Vertical earth pressure
$D$ = Outside diameter of pipe
$\gamma_w$= Unit weight of water
$h_w$= Height of water above pipeline

In most cases, the seismic hazards cannot be quantified precisely. Reasonable assumptions should be made to define the proper model for the seismic hazard.

When the pipeline is located below water table and placed in a trench, the vertical earth pressure on the pipeline can be calculated as

$$P_v = \gamma_w h_w + R_w \gamma_c C$$

Where

$R_w$ =A factor for water buoyancy = $1 - 0.33\left(\frac{h_w}{C}\right)$
$C$= Height of soil fill over pipeline
$\gamma_d$= Dry unit weight of backfill
$h_w$ = Height of water over pipeline

The extent of liquefaction = $L_b = 40m$

$$F_b = \frac{\pi D^2}{4}(\gamma_{sat} - \gamma_{content}) - \pi D t \gamma_{pipe} = \frac{\pi \times 0.506^2}{4}(18000 - 1000) - \pi \times 0.506 \times 0.004 \times 78560$$
$$= 2919\, N/m$$

The bending stress in the pipeline due to the uplift force ($F_b$) can be calculated as

$$\sigma_{bf} = \pm \frac{F_b L_b^2}{10Z}$$

Where

$L_b$ = Length of pipe in buoyance zone
$Z$ = Section modulus of pipe cross section = $\frac{\pi}{32} \frac{(0.506^4 - 0.498^2)}{0.506} = 0.000785 m^4$

$$\sigma_{bf} = \pm \frac{2919 \times 40^2}{10 \times 0.000785} = 595 \times 10^6\, N/m^2$$

Maximum strain in the pipe corresponding to the above bending stress can be evaluated as

$$\varepsilon = \frac{\sigma_{bf}}{E}\left[1 + \frac{n}{1+r}\left(\frac{\sigma_{bf}}{\sigma_y}\right)^r\right] = \frac{595\times10^6}{2\times10^{11}}\left[1 + \frac{9}{1+10}\left(\frac{595\times10^6}{358\times10^6}\right)^{10}\right] = 0.0039 = 0.39\% \text{ (tensile/compressive)}$$

The operational strain in the pipeline = $\varepsilon_{oper} = 0.000062$

The allowable strain in pipe in tension is =3% =0.03

The allowable strain in pipe in compression is $\varepsilon_{cr-c} = 0.175\frac{t}{R} = 0.175\frac{0.004}{0.253} = 0.0028 = 0.28\%$

The maximum strain in the pipeline due to buoyancy effect is less than the allowable strain for steel pipes in tension and compression.

# 8 Fault Crossing

The expected normal-slip fault displacement = $\delta_{fn} = 2.5m$

Dip angle of the fault movement = $\psi = 35^o$

The angle between the pipeline and fault line = $\beta = 40^o$

Component of fault displacement in the axial direction of the pipeline

$$\delta_{fax} = \delta_{fn} cos\psi sin\beta = 2.5\times cos35^o\times sin40^o = 1.32m$$

Component of fault displacement in transverse direction of pipeline

$$\delta_{ftr} = \delta_{fn} cos\psi cos\beta = 2.5\times cos35^o\times cos40^o = 1.57m$$

Important factor for the pipeline class-III = $I_p = 1.0$

The design fault displacement in axial direction = $\delta_{fax-design} = \delta_{fax}\times I_p = 1.32\times1 = 1.32\ m$

And the design fault displacement in transverse direction= $\delta_{ftr-design} = \delta_{ftr}\times I_p = 1.57\times1 = 1.57\ m$

The average pipe strain due to fault movement in axial direction can be calculated as

$$\varepsilon = 2\left[\frac{\delta_{fax-design}}{2L_a} + \frac{1}{2}\left(\frac{\delta_{ftr-design}}{2L_a}\right)^2\right]$$

Where

$L_a$ = effective unanchored length of the pipeline in the fault zone

$$L_a = \frac{E_i\varepsilon_y\pi Dt}{t_u} = \frac{2\times10^{11}\times0.002\times\pi\times0.506\times0.004}{52585} = 49m$$

Or

$L_a$=the actual length of the anchorage = 100 m

Hence,

The anchored length to be considered is the lower of the above two values. So $L_a$=49m

Hence the axial strain in the pipe = $\varepsilon = 2\times\left[\frac{1.32}{2\times49}+\frac{1}{2}\left(\frac{1.57}{2\times49}\right)^2\right] = 0.027 = 2.7\%$ (tensile)

The operational strain in the pipeline = $\varepsilon_{oper} = 0.000062$

Hence the total strain in the pipeline in tension is = 0.027+0.000062=2.7%

The allowable strain in pipe in tension is = 3% = 0.03

The total tensile strain in the pipe due to fault crossing is less than the allowable strain.

# 9 Seismic Wave Propagation

The expected peak ground acceleration of the site (take Christchurch for example) = $PGA_r = 0.34g$ (site class D or E, AEP =1/10)

For soil class D or E, the peak ground acceleration (PGA) at ground = $0.34g\times I_g = 0.34g\times1.2 = 0.41g$

Converting PGA to PGV using the correlation (ALA 2001; ALA 2005), considering soft soil, with source-to-site distance less than 20 km, $M_w$ = 7.5 (representative 2010-2011 Christchurch Earthquake Sequence)

$$\frac{PGV}{PGA} = 208$$

Hence $PGV = 0.41g\times208 = 85.3cm/s$

Design peak ground velocity =$V_g = PGV\times I_p = 85.3\times1.0 = \frac{85.3cm}{s} = 0.85m/s$

Maximum axial strain in the pipe due to wave velocity can be calculated as

$$\varepsilon_a = \frac{V_g}{\alpha_\varepsilon C} = \frac{0.85}{2\times2000} = 0.00021$$

Where

$C$ = Velocity of seismic wave propogation= 2km/s

$\alpha_\varepsilon$ = Ground strain coefficient = 2.0 (for S-wave)

Maximum axial strain that can be transmitted by soil friction can be calculated as

$$\varepsilon_a = \frac{t_u\lambda}{4AE} = \frac{52585\times1000}{4\times0.0063\times10^{11}} = 0.02$$

Where

A = Cross section area = $\frac{\pi}{4}(0.506^2 - 0.498^2) = 0.0063m^2$
$\lambda$ = Apparent wave length of seismic waves at ground surface = 1000m

# 10 Conclusion

The calculation above were completed for a buried continuous PVC/Welded steel (DN450 Steel pipe or DN300 Class C PVC pipe) pipeline which is designed to dispatch water at a pressure of 650 KPa. It was calculated that:

- The calculated axial strain due to wave propagation (0.00021) needs not be larger than the strain transmitted by soil friction (0.02)
- The operational strain in the pipeline = $\varepsilon_{oper}$ = 0.000062
- Hence the total strain in pipe in tension = 0.00021+0.000062=0.000272 = 0.272%
- The allowable strain in pipe in tension is = 3% = 0.03

This proved that the maximum strain in pipe due to wave propagation is less than the allowable strain.

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