# THE RATIONAL METHOD – FREQUENTLY USED, OFTEN MISUSED

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## ABSTRACT

The Rational Method has been in use in some form or another at least since 1889, where its use appeared in a paper by Kuichling titled "The Relation between Rainfall and the Discharge in Sewers in Populous Districts". It is not so much that the method itself is judged as being rational, but more than the method relates to the ratio between rainfall and runoff (which is essentially the runoff coefficient, C).

The method enjoys a prominent place in flow estimation guidelines in New Zealand and internationally. Unfortunately it is frequently misapplied, possibly as a consequence of its apparent simplicity. There are (erroneous) perceptions that the method can be used to estimate discharge for a wide variation in rainfall duration, to estimate total runoff volume and hence for sizing of mitigation works, and for determination of flood hydrographs for unsteady analyses.

In this paper the limitations of the method are examined in detail, with the intention of clarifying its use and range of applicability. Detail into the derivation of runoff coefficients that are applied is given, and some less-widely known applications of the method (including the "probabilistic approach") are described. This paper is intended for a practitioner audience.

## KEYWORDS

**Rational Method, Rainfall-Runoff, Peak Flow Estimation**

## PRESENTER PROFILE

Mark is a Civil Engineer with some 18 years of post-graduate experience, the majority of which has been spent in hydrological and hydraulic investigations and analyses. He has a Master's degree in hydraulics and his main focus in the last few years has been in urban stormwater management and in flood management for river systems.

## 1 INTRODUCTION

The Rational Method is widely publicised as a simple and effective method for use in hydrological calculations. Published data exist that cover a wide range of applicability, and it appears as a published method in many guideline and regulatory documents.

A "rational" number is defined mathematically as one that can be expressed as a *ratio* of two integers. For example, the number 3/4 (0.75) is a rational number while √2 (1.4142...) is not a rational number. Considering the antonyms for the word "rational", the terms "absurd", "irrational" and "non-sensical" are given by at least one popular dictionary. Consequently it is hoped and assumed that *every* method in common use for rainfall-runoff analysis is *a* "rational method", but not necessarily *the* Rational Method.

The notion of the term "rational" being a reference to a ratio is described further in this paper.

While many guideline documents give a full description of the Rational Method and its use in peak flow estimation, it is frequently the designer's objective to size mitigation works. The Rational Method can be used to give an indication that changes in land use result in changes to hydrological response, but its application to quantifying these changes is somewhat limited. Many guideline documents miss this.

# 2 RATIONAL METHOD THEORY

The Rational Method Formula is given as

$$q = F.C.i.A$$

Where $q$ = peak discharge $[L^3/T]$
$F$ = units conversion factor
$C$ = dimensionless runoff coefficient
$i$ = rainfall intensity for duration equal to catchment time of concentration [L/T]
$A$ = catchment area $[L^2]$

Examination of this formula reveals that the product of rainfall intensity and catchment area has unit equivalent to that of peak discharge $[L^3/T]$. Therefore it can be seen that the rate of "inflow" to the catchment is given by $i.A$ (and is a steady rate over rainfall duration). In response to this, the peak rate of outflow $q$, is given by the formula. Thus for a consistent set of units (where $F = 1$), runoff coefficient $C$, represents a ratio between inflow and outflow.

$$C = \frac{q}{i.A} = \frac{out\_flow}{in\_flow}$$

As an example, a runoff coefficient, $C$, value of 0.8 can be taken to mean that the peak rate of discharge from a catchment is 80% of the average rate of rainfall accumulation in the catchment. In consideration of this, it may appear odd that peak outflow rate should be linked by a constant to average inflow rate. During a rainfall-runoff process, it would be usual for runoff (i.e. outflow) to begin at zero in response to rainfall (inflow), and gradually increase with continuous rainfall to a point at which outflow equals inflow (i.e. it tends towards a steady state). Given sufficient rainfall, therefore, the runoff coefficient, $C$, should tend towards a value of unity and not be limited to published $C$-values.

In general rainfall depth-duration-frequency data will show decreasing intensity with increasing duration. Selection of a rainfall intensity corresponding to duration greater than time of concentration (even with the same runoff coefficient) will result in a peak discharge estimate that is lower than what would be obtained if using the (higher) intensity that would correspond to duration equal to time of concentration.

This emphasises the importance of the duration that is applied to a Rational Method analysis. From a theoretical perspective, prescribed runoff coefficients are generally "calibrated" for use only when rainfall duration exactly equals catchment time of concentration. When duration is less than this, not all of the catchment is able to contribute runoff and the catchment area, $A$, should be adjusted in the formula to reflect this. When duration exceeds time of concentration the runoff coefficient to be applied should begin to approach a value of 1 with increasing duration. Therefore the runoff coefficient is only applicable to a rainfall duration that is equal to catchment time of concentration.

# 3 EXAMPLE PROBLEM

For the purpose of demonstration, an example problem has been set up and will be referred to throughout this paper. The example is a small undeveloped catchment that is set to be developed for residential purposes, shown in Figure 1. The example catchment is rectangular in shape, covering 10 hectares in area with uniform slope of 1:50. A collector channel collects sheet flow runoff and delivers this to the observation point.

The Rational Method has been used to estimate the hydrological response for both the pre- and post-development scenarios, with calculations summarised in Table 1.Time of concentration is calculated by published methods, and the result is used in selection of design rainfall intensity to be applied from depth-duration-frequency tabulated values. The Rational Method "C" value (often termed "runoff coefficient") is selected from standard published data.

*Figure 1: Example Catchment*

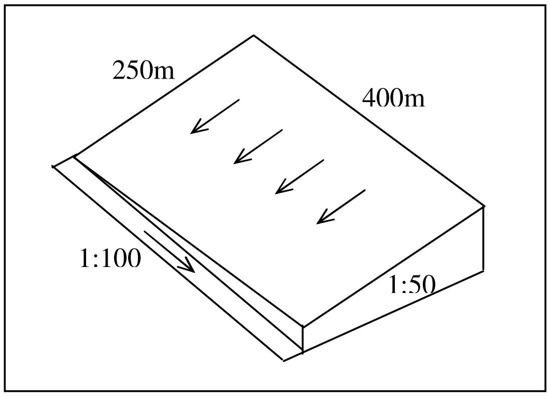
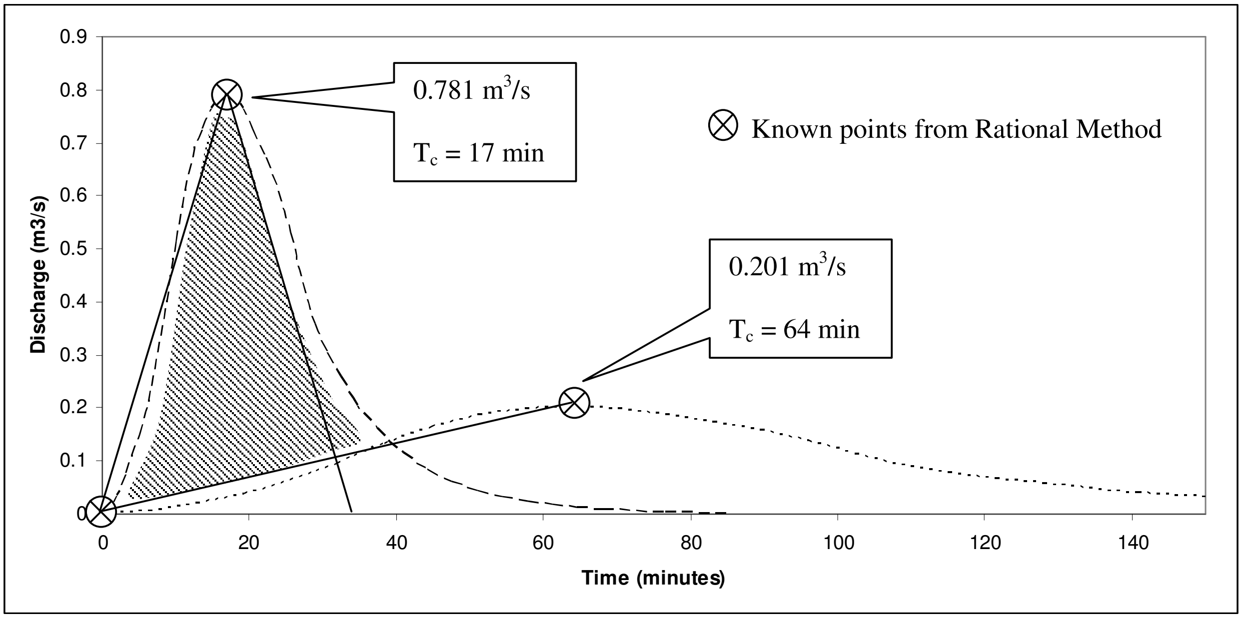


*Table 1: Results of Analysis for Example*

| Catchment Condition | Time of Concentration (min) | Design Rainfall Intensity (mm/h) | Rational Method "C" | Peak Flow Estimate ($m^3/s$) |
|---|---|---|---|---|
| Pre-Development | 64 | 24.1 | 0.30 | 0.201 |
| Post-Development | 17 | 51.1 | 0.55 | 0.781 |

From the results shown in Table 1 it can be seen that development of the currently undeveloped catchment will have a notable change on the hydrological response (higher peak discharge and shorter response time). The Rational Method has been appropriately applied in this case to estimate the peak discharge for each case. However the results reveal differences in performance that are difficult to compare directly as the peak discharge estimates apply to two different rainfall durations. These results are sketched schematically in Figure 2. In this the rising and falling limbs of each hydrograph are shown dashed as the analysis does not give any detail on these. Rather, just two points on each hydrograph are given by the Rational Method, these being zero discharge at time equals zero, and peak discharge at time equals time of concentration. Any further information shown in Figure 2 is surmised in this case, and cannot be used for more detailed analysis or optioneering.

*Figure 2: Analysis Results*



The problem frequently facing designers is in what measures should be put in place to mitigate the effects of the development, and this is where use of the Rational Method is frequently stretched, often into misuse. Some common misuses include the following:

1. Use the Rational Method to determine peak discharge estimates for a range of rainfall durations such that a comparison between pre- and post can readily be made. For example, for the pre-development case, find out the peak flow in response to a 17-minute event and compare against the post-development peak flow estimate.

2. Using the "runoff coefficient" (C-value), estimate total runoff volume for the design events and provide storage for the difference to mitigate effects. For example, with C = 0.30 this means that 30% of total rainfall onto the catchment eventually runs off the catchment, and comparison of this volume with a similarly calculated volume for the post-development case yields a storage volume that will adequately mitigate effects of development.

3. As the average rainfall intensity was used in the calculation, this gives average runoff rate over the rainfall duration. Total volume for each catchment condition can therefore be calculated by finding the product of flow rate and duration, with the difference between these being required as storage for mitigation of effects.

4. By application of an "appropriate" or "generic" hydrograph shape, find the volume represented by the shaded area in Figure 2, provide this while constraining outflow to pre-development peak rate and this represents adequate mitigation of adverse effects resulting from the changed hydrological response.

The above bullet points are all incorrect applications of the Rational Method. Specific investigation into these misuses will is given later in the next section.

# 4 APPLICATION OF THEORY TO THE EXAMPLE

The example referenced above has been used to demonstrate a typical application of the Rational Method where inappropriate analyses are frequently encountered. The problem here is to size mitigation measures to reduce the impact of a change in land use on the rainfall-runoff response from a catchment.

## 4.1 CHANGING RAINFALL DURATION

The first bullet point in Section 3 of this paper indicates use of the Rational Method for rainfall duration being something different to catchment time of concentration. The effects of this are outlined below.

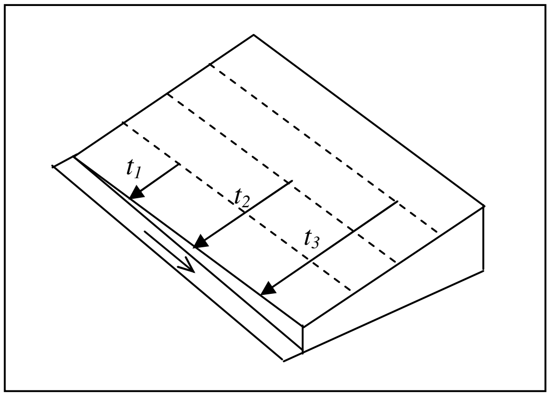
In the pre-development catchment condition, the contributing catchment area for rainfall duration less than time of concentration can be calculated using the Manning equation.

$$t = \frac{100.n.L^{0.33}}{s^{0.2}}$$

Where $t$ = travel time (minutes) [T]
$n$ = Manning roughness coefficient [$L^{-1/3}$ .T]
$L$ = up-slope length of contributing catchment (m) [L]
$s$ = catchment slope in %

Knowing time $t$ the above equation may be solved for $L$ which can be used to calculate contributing catchment area $A$. In this way the catchment may be split into sub-areas by isochrones, along which overland travel time is constant (as shown in Figure 3). Thus for each rainfall duration that is less than catchment time of concentration, a different contributing area (that is less than total area) should be used.

*Figure 3: Isochrones in Example Catchment*



For $t$ = 17 minutes, this equation may be solved for $L$ to yield $L$ = 7.45m, which gives a catchment area $A$ = 2,980 $m^2$.

Using this in the Rational Method Formula yields $q = 0.013\ m^3/s$.

This number may be compared to the post-development rate of 0.781 $m^3/s$, but as these are for rainfall events of vastly different contributing catchment area, such a comparison is not meaningful for the purpose of sizing mitigation works. Should consistent catchment areas be used then runoff coefficient should not be the same if duration is kept constant, rendering use of the Rational Method here for direct comparison to be somewhat meaningless.

## 4.2 USING THE RUNOFF COEFFICIENT TO FIND RUNOFF VOLUME

As has been explained above, the runoff coefficient, *C*, represents a simple ratio between inflow and outflow that has been observed to occur over various surface types when rainfall duration and catchment time of concentration are equal. During the period of time between the onset of rainfall and the catchment time of concentration, the accumulation of runoff volume is very unlikely to be linear, meaning that a constant runoff coefficient over this time does not apply. Loss models used in hydrological simulation generally all agree with this, where hydrological losses tend to decay with time.

Using the runoff coefficient as a volumetric runoff coefficient as alluded to in bullet point 2 above, results using the example referenced above yields the runoff volumes given in Table 2.

*Table 2: Runoff Volume Estimates*

| Catchment Condition | Rainfall Duration (min) | Design Rainfall Intensity (mm/h) | Rational Method "C" | Runoff Volume Estimate ($m^3$) |
|---|---|---|---|---|
| Pre-Development | 64 | 24.1 | 0.30 | 771.2 |
| Post-Development | 17 | 51.1 | 0.55 | 796.3 |

The above volume estimates appear to reveal no great change to runoff volume resulting from development of the example catchment. While such comparisons are frequently encountered, it is not meaningful to compare runoff volumes from rainfall events of such differing duration. However, as mentioned previously, the Rational Method cannot be used for duration that is different from catchment time of concentration. Furthermore, the above calculation is based on the assumption that Rational Method runoff coefficient represents a volumetric runoff coefficient, which is clearly false.

## 4.3 PRODUCT OF DISCHARGE AND DURATION TO FIND RUNOFF VOLUME

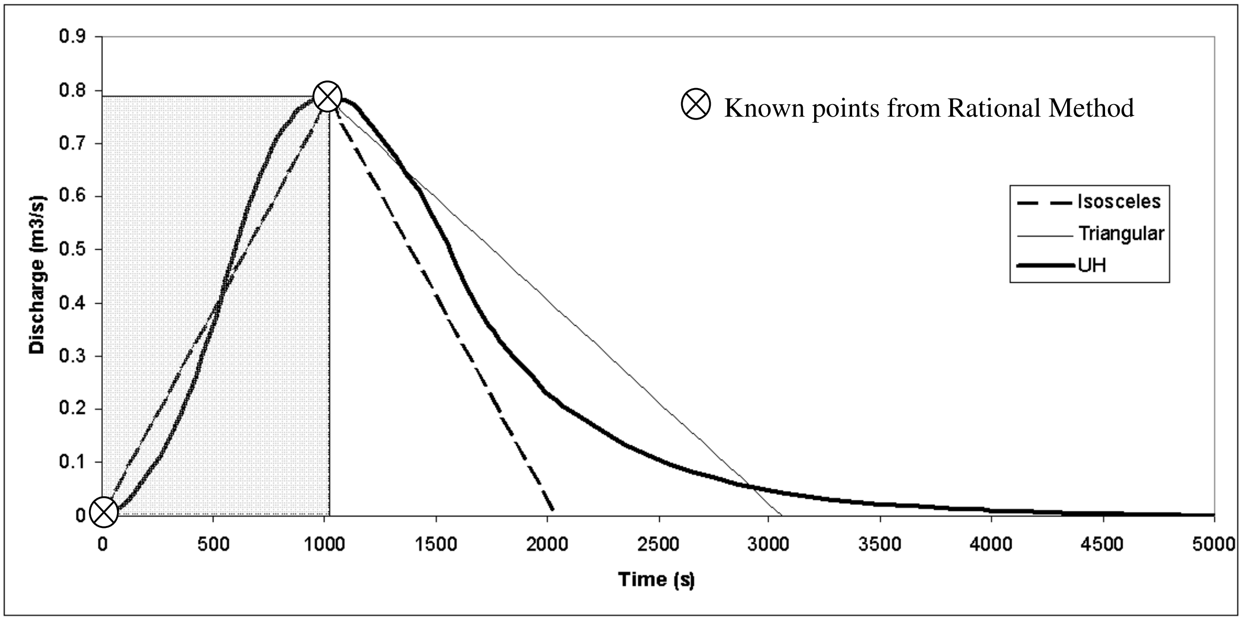
Instead of attempting to use the runoff coefficient for volume estimation, total runoff volume can be given by the integration of discharge with time. Where discharge is steady, this can be simplified to the product of discharge and time. However, application of the Rational Method Formula only gives just two points on the runoff hydrograph. These points are plotted at zero discharge for time equals zero, and peak discharge at time equals catchment time of concentration. The Rational Method does not provide any further information on hydrograph shape.

In Figure 4 a series of runoff hydrographs are shown. All of these have the same peak discharge at the same time of concentration, but clearly different total runoff volume (area under the curve). One hydrograph has equal time for rising and recession limbs, and is represented by an isosceles triangle. Another shows the shape if the recession time is twice that of the time to peak, making a triangular shape, and lastly the SCS Unit Hydrograph shape has been fitted to the data (curvilinear plot). Also shown (shaded) is the result that would occur if volume we calculated by finding the product of peak discharge as given by the Rational method Formula and time of concentration.

From inspection it can be seen that if the isosceles triangle hydrograph shape is correct (i.e. representative of the true catchment response), then the volume estimate given by the product of peak discharge and catchment time of concentration is exactly correct. However for the other hydrograph shapes shown, volume estimates will be low.

The Rational Method only provides two data points on each hydrograph, as shown in Figure 4, and therefore its use for estimation of total runoff volume is limited.

*Figure 4: Hydrograph Shapes*



In the above case, the product of discharge and duration (or catchment time of concentration) yields a volume estimate of 796.6 m$^3$/s. A larger volume would result if one of the other hydrographs plotted in Figure 4 was used, and it is likely that the volume estimate of 796.6 m$^3$/s would be close to a lower bound. A similar calculation for the pre-development case yields a total runoff volume of 771.8 m$^3$/s, with the difference between these two (some 25 m$^3$) being largely meaningless as it has been calculated by comparison between results from two very different scenarios.

To generate an accurate runoff hydrograph for volume estimation a temporal rainfall distribution, or hyetograph, is required, the use of which is outside of the range of applicability of the Rational Method.

## 4.4 HYDROGRAPH VOLUME DIFFERENTIAL

Bullet point 4 in Section 3 suggests an approach whereby mitigation volume is estimated by integration of discharge hydrographs with time and differencing the pre- and post-development values. This is a valid approach, but difficulty lies in its dependence on assumed hydrograph shape. Examination of Figure 2 suggests that a reasonable approximation may be possible by making an assumption of triangular hydrographs to find this difference. The assumption of the rising limb being linear on each of the pre- and post-development hydrographs is likely to be reasonably representative, however the falling limb slope is strongly dependent on individual catchment characteristics. The differencing approach using triangular hydrographs relies, in this case, on the falling limb of the post-development hydrograph and the rising limb of the pre-development hydrograph both being linear.

Furthermore, the recession time assumed in plotting triangular hydrographs becomes relevant to this calculation.

For the purpose of comparison, an isosceles triangle hydrograph shape approximation for both pre- and post-development hydrographs will result in a required detention volume of some 695 m$^3$ for the example in this paper. It is reasonably plain to see that if recession time for the post-development hydrograph in Figure 2 were extended out to be greater than time to peak, a greater volume estimate would result from application of this method.

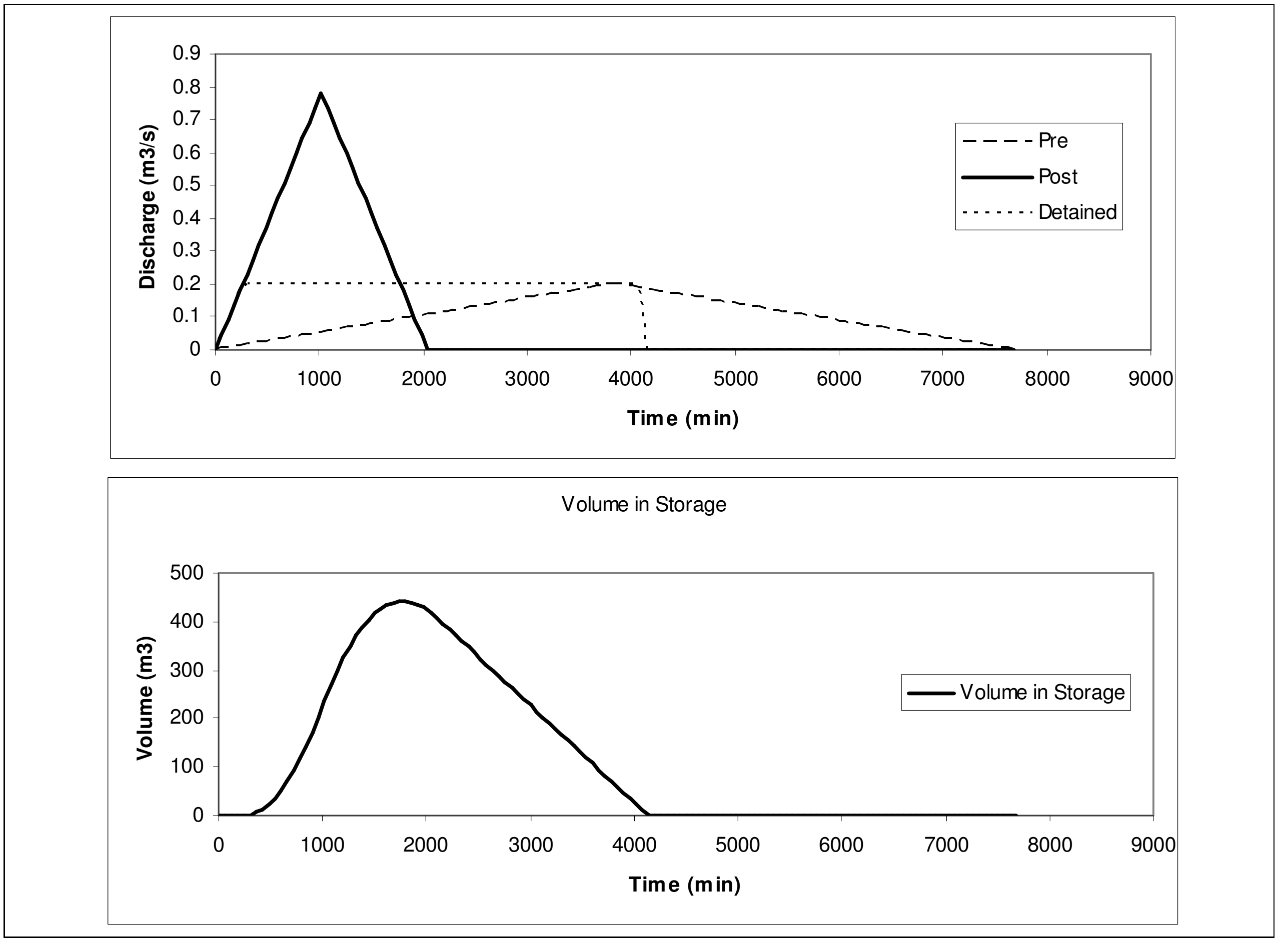
## 4.5 TYPICAL SOLUTIONS USING THE RATIONAL METHOD TO DETERMINE DETENTION VOLUME

In the example described above, it is often assumed (sometimes incorrectly) that adequate mitigation has been provided if the post-development peak discharge is constrained to no more than that for the pre-development case. The reason that mitigation sized in this way is not necessarily adequate is because differing downstream flood mechanisms and conditions may be present that are not all peak discharge sensitive. In many instances, prolonging the duration over which a threshold discharge is attained may increase stream erosion, or may exacerbate flooding where capacity constraints exist.

However, these effects are ignored in this paper for the purpose of demonstration. Three different hydrograph shapes have been used in determination of required detention volume if post-development peak discharge is to be constrained to no more than the pre-development rate. These three shapes are those shown in Figure 4. In each case required detention volume has been calculated by an algorithm that allows maximum flow (at pre-development rate) to occur at all times, with the difference between inflow and maximum allowable outflow being taken to storage. Storage volume is released such that the maximum outflow never exceeds the pre-development peak discharge. This is not entirely realistic as the outflow configuration and detention structure shape will result in different performance in reality, but this analysis represents a lower bound and a basis on which to conduct comparisons. In most cases outflow rate will not be able to be kept constant over a range in detained volume due to changes in hydrostatic head through an outlet structure.

In Figure 5 the results are shown if both pre- and post-development hydrograph shapes, with peak and time-to-peak determined using the Rational Method, are assumed to be representative of the catchment response.

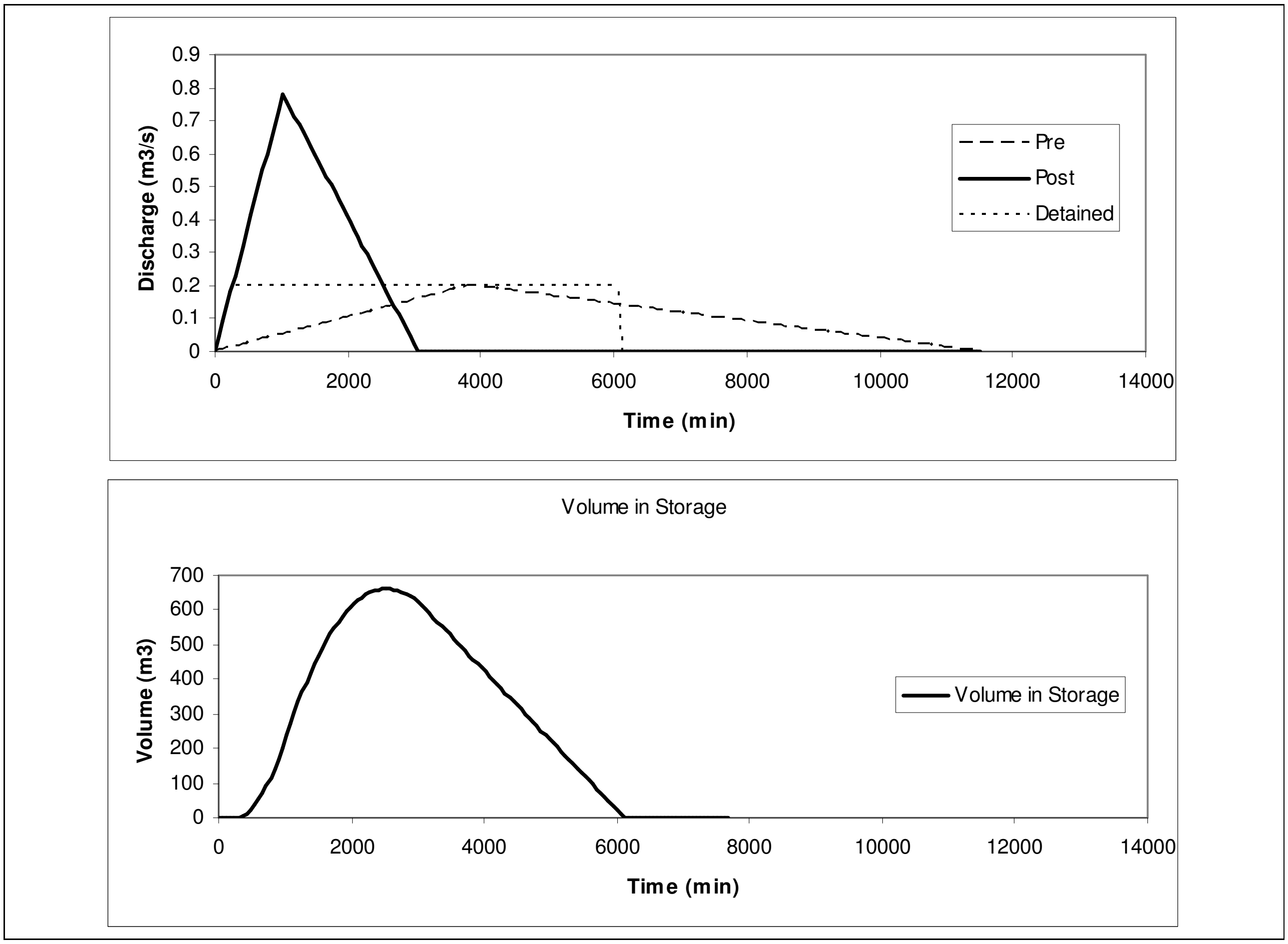
*Figure 5: Results Assuming Isosceles Hydrographs*



The result is a required detention volume of 440.5 $m^3$.

In Figure 6 the same analysis is applied to similarly developed hydrographs, except that the triangular shape of these differs in that the recession time is twice the time-to-peak, for both pre- and post-development cases. Clearly in this case the total runoff volume is much larger than that which would be given by the hydrographs shown in Figure 5.

*Figure 6: Results Assuming Triangular Hydrographs*

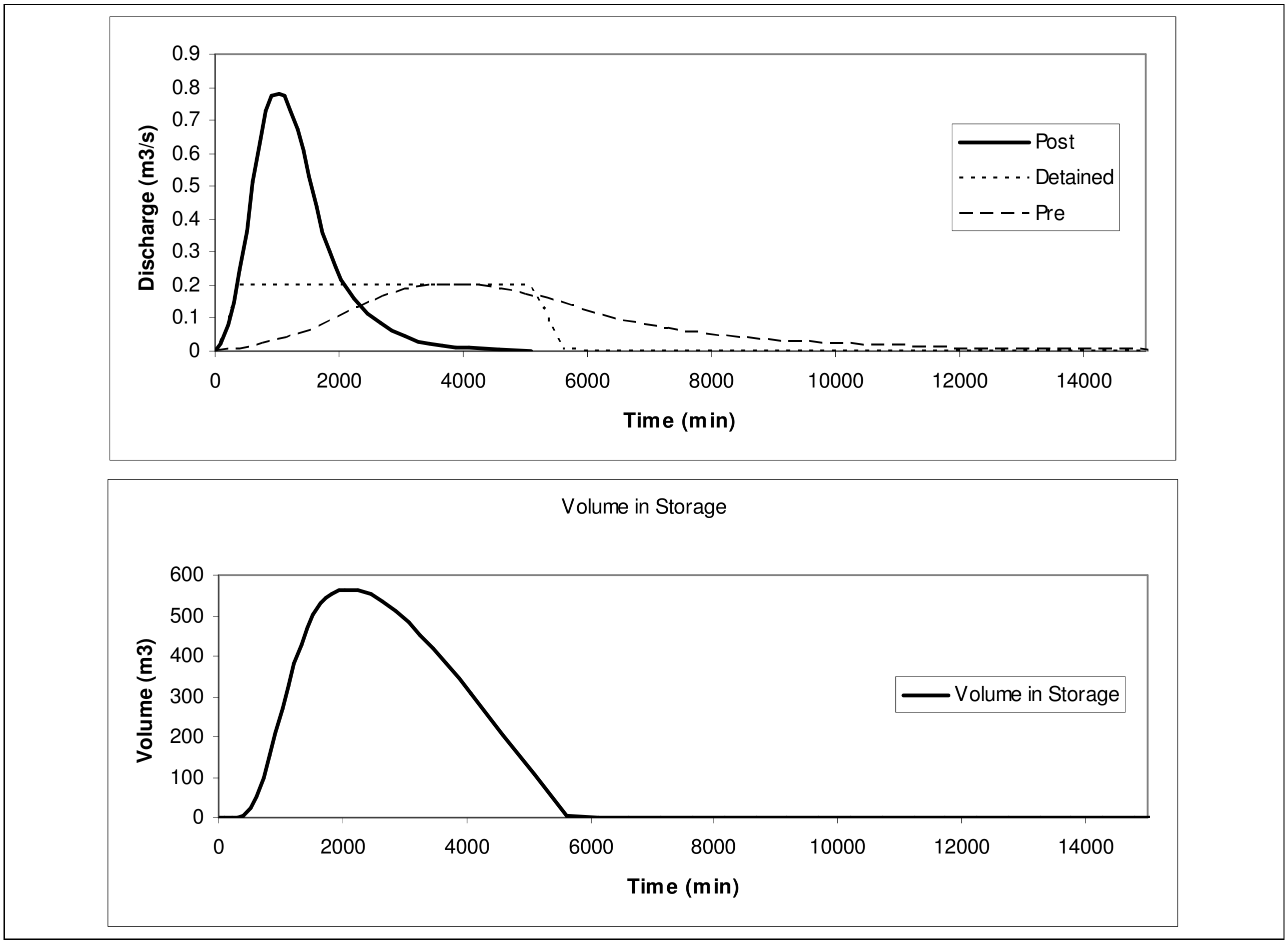


The result is a required detention volume of 660.3 $m^3$.

In Figure 7 a similar pair of plots are presented, generated this time using the shape of the SCS Unit Hydrograph, well-known to many.

The resulting requirement for detention is 562.8 $m^3$.

*Figure 7: Results Assuming SCS Unit Hydrograph Shape*



Thus it can be seen that hydrograph shape, which is not provided by the Rational Method, is of fundamental importance to the calculation of mitigation works. As alluded to in the introduction to this paper, the Rational Method is frequently put forward in guideline documents as a method by which flow estimates can be made. This is entirely correct. What is often missing, however, is that most assessments in compliance with guideline documents are conducted with the purpose of sizing mitigation works. It is here that the Rational Method has to be used with extreme care, and is often insufficient for the stated purpose.

For the specific example used in the analysis above, a range in required detention volume from 440 $m^3$ to 660 $m^3$ can be obtained (a 50% range), depending on the assumption of hydrograph shape. As significant sensitivity to this parameter (hydrograph shape) is shown, an appropriate approach would be to achieve greater accuracy in this for the analyses to be conducted. This greater accuracy in hydrograph shape is something that cannot be provided by the Rational Method, but rather a more detailed alternative approach would be required. This could involve modelling, which would introduce a further parameter in that of temporal rainfall distribution that would require accurate definition.

# 5 PROBABILISTIC APPROACH

The Rational Method can be used in a probabilistic approach. The Rational Method Formula may be re-written as

$$q(Y) = F.C(Y).i(t_c, Y).A$$

In the above formula *C, q,* and *i* are labelled with average recurrence interval *Y* years. Using this approach it is not runoff in response to a particular rainfall event that will be the desired outcome. Rather, the intention is to use this approach to estimate discharge for a particular ARI by frequency analysis of observed data.

In application of this method, data requirements include frequency curves of both rainfall of duration equal to $t_c$ and corresponding discharge. If both *q* and *i* are known, the equation allows solution for runoff coefficient *C.*

The relevance to this paper is that the runoff coefficient, *C*, may not be constant across events of differing ARI. This is recognised in some guideline documents, but not in others. In general *C* increases with increasing ARI. This method is fully described in Maidment (1992) and is not repeated here.

# 6 CONCLUSIONS

The Rational Method is widely prescribed and recommended for use in peak flow estimation, and has been shown to yield results of acceptable accuracy if used appropriately.

The key parameter in the Rational Method Formula is that of the coefficient *C*, tabulated values of which appear in many reference texts. These values have been explicitly derived for use when rainfall duration exactly equals catchment time of concentration. Use of published values for *C* under different conditions is likely to be erroneous.

The Rational Method gives a ratio of inflow to outflow, under the specific conditions of rainfall duration equal to catchment time of concentration.

The coefficient *C* has been shown to vary both with rainfall duration and with event severity (i.e. ARI).

It is difficult to make use of the Rational Method results to estimate detention storage, without making an approximation on hydrograph shape.

Hydrograph shape for any catchment is dependent on temporal variation in rainfall and also on specific catchment characteristics, and it is difficult to conclude that a single shape should be representative of all catchments.

## REFERENCES

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