Percentile Standards: Rationale, Implementation and a Few Misconceptions

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Colours

- Key terms, blue
- Bad ideas, red
- Good things, green

Why consider statistics now?

- To put a monitoring-for-compliance on a sound footing
- Identify efficient and transparent approaches
- Avoid misconceptions
  - e.g., need at least 19 samples to estimate a 95%ile
- Examine and cope with “tricky issues”
  - rolling compliance assessment periods (drop a month, add another)
- “The statistically correct method” doesn’t exist
  - Need to identify the most appropriate and consistent one
  - Plenty of inappropriate ones!
- Compliance assessment is a statistical issue (among others)

95th percentile vs. 95% confidence

- Can get confusing (two “95”s)! Just know that we can have

  1) The 95th percentile (= “95%ile”)
     - The value of a discharge’s water quality variable that is not exceeded for 95% of the time

     or, equivalently

     - The value of a discharge’s water quality variable that *is* exceeded for 5% of the time

  2) With 95% confidence
     - That the discharge’s 95%ile water quality limit is below the consent’s limit

Examples

- MoH Drinking-Water Standards' *E. coli* compliance rules are based on a requirement that sampling be sufficient to allow 95% confidence that a stated limit (zero *E. coli*) has not been exceeded for at least 95% of the time
  - or that the limit (zero *E. coli*) *has* been exceeded for no more than 5% of the time
- Those Standard's turbidity rules also use 99%iles. They are based on a requirement that sampling be sufficient to allow 95% confidence that a turbidity limit has not been exceeded for at least 99% of the time
  - or that the limit *has* been exceeded for no more than 1% of the time
- WaterNZ/MfE *New Zealand Municipal Wastewater Monitoring Guidelines,* Table 13.2. Based on 90% confidence (more later)

Why percentiles?

- For most water quality variables a feasible maximum will inevitably be exceeded once enough samples are taken
  - a deterrent to sampling
  - and if exceeded, so what?
- Medians are useful for stating expected performance of a WWTP in the normal-course-of-events
- Other percentiles (e.g., 90%ile, 95%ile, 98%ile) useful in accounting for unusual events
  - not a deterrent to sampling
    - estimated 95%iles tend to decrease with sample size.

Fundamentals

- A discharge percentile compliance rule should state:
  1. A compliance assessment period
  2. The sample statistics to be used
     - tolerance limit, percentile, number of exceedances,…
  3. Compliance definition (in terms of those statistics)

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  1. A compliance assessment period
  2. The sample statistics to be used
     - tolerance limit, percentile, number of exceedances,…
  3. Compliance definition (in terms of those statistics)
- The rule should be derived from percentiles *of time*
  - Statistics 101: Take *samples* to estimate the frequency characteristics of a *population*
  - Therefore must consider "statistical sampling error"
    - "*Slings and arrows of outrageous fortune*" (pers. comm., W. Shakespear)

So? (using 90%ile as an example)

- The compliance rule for a 90%ile discharge standard isn't necessarily based on the 90%ile of the samples
  - Only would do so if an "even-handed" burden-of-proof were adopted
  - Precautionary ⟹ higher sample percentile (minimise consumer's risk)
  - Permissive ⟹ lower sample percentile (minimise producer's risk)

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  - Permissive ⟹ lower sample percentile (minimise producer's risk)
- More formally (using 90%iles as an example)
  - Precautionary ⟹ compare discharge percentile limit with upper one-sided 95% tolerance limit
  - Permissive ⟹ compare discharge percentile limit with lower one-sided tolerance limit

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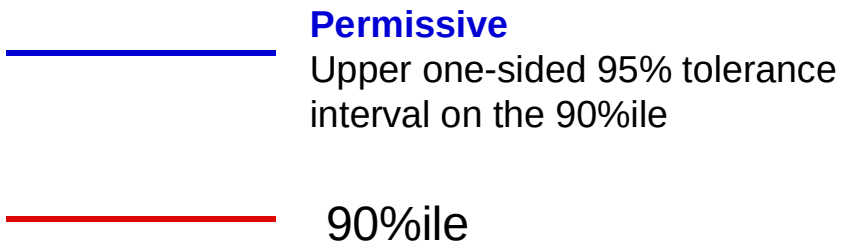

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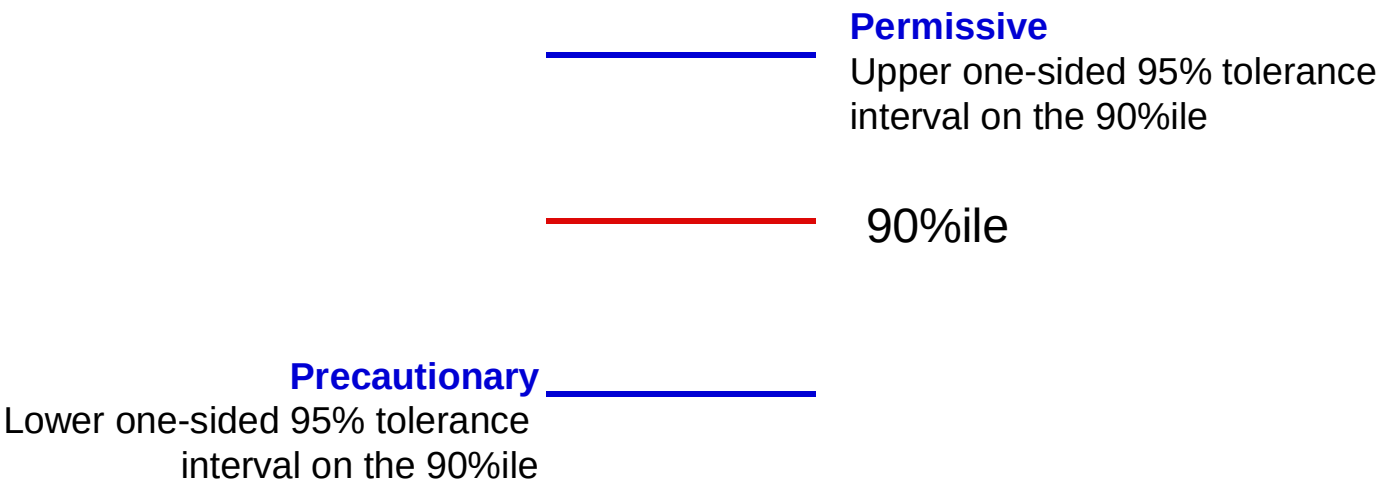

So? (using 90%ile as an example)

Permissive
Upper one-sided 95% tolerance interval on the 90%ile

90%ile

Precautionary
Lower one-sided 95% tolerance interval on the 90%ile

Interpretation

Permissive: I would only be 95% confident that the red line has been breached if calculations from my samples result in the upper blue line.

Precautionary: I would only be 95% confident that the red line has *not* been breached if calculations from my samples result in the lower blue line.

But…

- There are many ways to estimate percentiles
  - Excel's is very unusual
  - Most software manuals don't tell you what estimation method is used
- Can be issues in calculating tolerance limits on percentiles
  - Often based on assumptions that are hard to test
    - especially concerning the water quality variable's statistical distribution
  - Simpler methods can be "biased"
- You often don't know if a breach has occurred until the end of the compliance assessment period
  - Can't calculate the tolerance limit until all data are to hand

A better way

- Base the compliance rule on the number of exceedances of the discharge percentile limit from random sampling in an assessment period
  - No distributional assumptions needed
  - Probability that a single sample exceeds the percentile limit follows the binomial distribution
  - Simple!
- Next slide contains all the information used to develop the MoH *NZ Drinking-Water Standards* and the WaterNZ/MfE *New Zealand Municipal Wastewater Monitoring Guidelines*
  - I will demonstrate how that was done (simple reading of the appropriate graph)

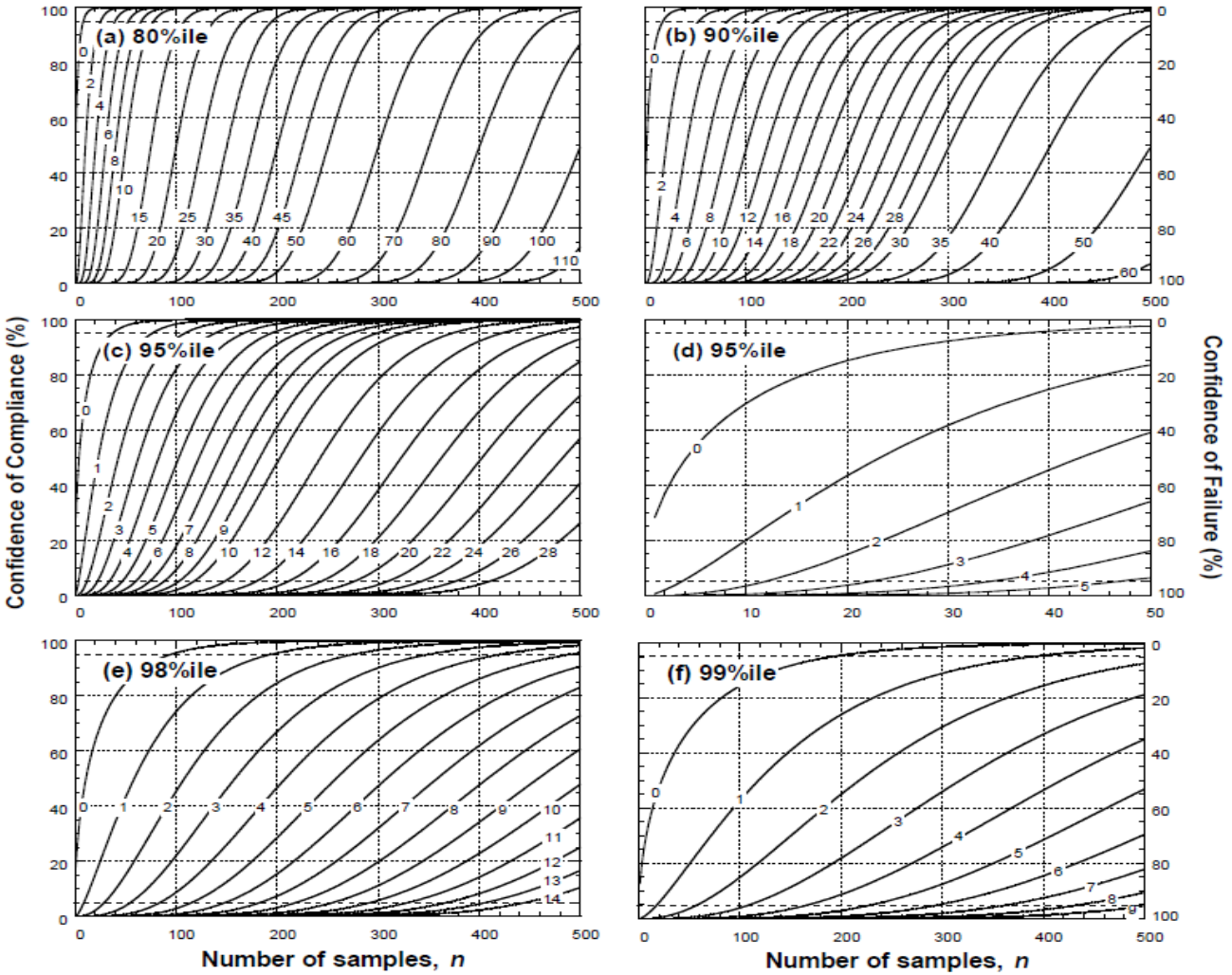

Fig. 8.5 Confidence of Compliance for five percentiles, using Jeffreys' prior. Numbers on the graphs are the observed number of exceedances. Note the condensed range on the horizontal axis for the 95%ile in part (d).

Sources: : McBride, G.B.; Ellis, J.C. 2001. Confidence of Compliance: a Bayesian approach for percentile standards. *Water Research 35(5)*: 1117–1124.
McBride, G.B. (2005). *Using statistical methods for water quality management*, Wiley, NY,

Drinking-water Standards 95%ile look-up table
Precautionary, minimises consumer's risk

Table A1.4: Allowable exceedences (for 95 percent confidence that the maximum acceptable value (MAV) is exceeded for no more than 5 percent of the time)

e	*n*	*e*	*n*	*e*	*n*	*e*	*n*
0	38–76	40	1025–1046	80	1908–1929	120	2773–2793
1	77–108	41	1047–1069	81	1930–1951	121	2794–2815
2	109–138	42	1070–1091	82	1952–1973	122	2816–2836
3	139–166	43	1092–1113	83	1974–1994	123	2837–2858
4	167–193	44	1114–1136	84	1995–2016	124	2859–2879
5	194–220	45	1137–1158	85	2017–2038	125	2880–2900
6	221–246	46	1159–1181	86	2039–2060	126	2901–2922
7	247–272	47	1182–1203	87	2061–2081	127	2923–2943
8	273–298	48	1204–1225	88	2082–2103	128	2944–2965
9	299–323	49	1226–1247	89	2104–2125	129	2966–2986
10	324–348	50	1248–1270	90	2126–2146	130	2987–3007
11	349–372	51	1271–1292	91	2147–2168	131	3008–3029
12	373–397	52	1293–1314	92	2169–2190	132	3030–3050
13	398–421	53	1315–1336	93	2191–2211	133	3051–3072
14	422–445	54	1337–1358	94	2212–2233	134	3073–3093
15	446–469	55	1359–1381	95	2234–2255	135	3094–3114
16	470–493	56	1382–1403	96	2256–2276	136	3115–3136
17	494–517	57	1404–1425	97	2277–2298	137	3137–3157
18	518–541	58	1426–1447	98	2299–2320	138	3158–3178
19	542–564	59	1448–1469	99	2321–2341	139	3179–3200
20	565–588	60	1470–1491	100	2342–2363	140	3201–3221
21	589–611	61	1492–1513	101	2364–2384	141	3222–3243
22	612–635	62	1514–1535	102	2385–2406	142	3244–3264
23	636–658	63	1536–1557	103	2407–2427	143	3265–3285
24	659–681	64	1558–1579	104	2428–2449	144	3286–3307
25	682–704	65	1580–1601	105	2450–2471	145	3308–3328
26	705–727	66	1602–1623	106	2472–2492	146	3329–3349
27	728–751	67	1624–1645	107	2493–2514	147	3350–3371
28	752–774	68	1646–1667	108	2515–2535	148	3372–3392
29	775–796	69	1668–1689	109	2536–2557	149	3393–3413
30	797–819	70	1690–1711	110	2558–2578	150	3414–3434
31	820–842	71	1712–1733	111	2579–2600	151	3435–3456

Source. MoH (2005). *Drinking-water Standards for New Zealand 2005 (Revised 2008)*. MoH.
http://www.health.govt.nz/publication/drinking-water-standards-new-zealand-2005-revised-2008-0

Municipal Wastewater Monitoring Guideline look-up table

Permissive, minimises producer's risk

Table 13.2: Number of exceedances (e) out of n samples permitted to meet percentile discharge compliance standards based on a discharger's risk of no more than 10%.

Number of samples taken in monitoring period (*n*)	Number of permitted exceedances (e) for a 10% discharger's risk to meet the performance standards listed.			
	Median (50%ile)	80%ile	90%ile	95%ile
5	4	2	1	1
6	5	2	2	1
7	5	3	2	1
8	6	3	2	1
9	6	3	2	1
10	7	4	2	1
11	8	4	2	2
12	8	4	3	2
13	9	4	3	2
14	9	5	3	2
15	10	5	3	2
16	11	5	3	2
17	11	6	3	2
18	12	6	3	2
19	12	6	4	2
20	13	6	4	2
21	13	7	4	2
22	14	7	4	2
23	15	7	4	3
24	15	7	4	3
25	16	8	4	3
26	16	8	5	3
27	17	8	5	3
28	17	8	5	3
29	18	9	5	3
30	19	9	5	3
31	19	9	5	3
32	20	9	5	3
33	20	10	6	3
34	21	10	6	3

Source: NZWERF/MfE. *New Zealand Municipal Wastewater Monitoring Guidelines*.
http://www.nzwwa.org.nz/Category?Action=View&Category_id=106#guidelines

Take-home messages

- Percentile standards' compliance rules should be derived from requirements on percentiles of time
- Simplest compliance rules are based on number of exceedances of a percentile limit in a required number of samples in an assessment period
- You must adopt (and should state) a burden-of-proof
  - permissive, even-handed, precautionary
- Can infer breach before the end of the assessment period
- Simple! (Lookup tables and graphs)